Linear Dissipation Models

"Nature has a great simplicity and therefore a great beauty."
Richard P. Feynman

The development of statistical models for linear dissipative systems was a major scientific achievement of the past century, in large part because they could be based on the concept of arbitrarily small perturbations of a Boltzmann distribution. Ohmic conductivity expressions with *kT* factors are but one example. Most are acquainted with expressions $V = I * R$, $W = I * V = I^2 * R = V^2 / R$, *etc.* Consider the lowly incandescent bulb rated *25W @ 120V.* One would calculate a resistance of *576 ohms*. But, should one measure it with an ohmmeter one might find *43 ohms*. Tungsten is a remarkably nonlinear material, yet the bulb's properties are also remarkably independent of it's history, excessive potentials precluded. It has no memory of how it got to the state it's in. This is characteristic of a thermodynamic steady-state. In thermodynamics, watts express the dissipation or rate of a system's loss of free energy. Dissipation depends only on boundary parameters and is a minimum with respect to internal perturbations with these parameters fixed.

A typical black box model might presume a specified set of boundary temperatures leading to a reproducible set of boundary fluxes independent of the manner in which contacts are applied. The fluxes themselves reflect properties of the box's contents. For two thermal contacts,

$$W = (J/T_1) * (T_1 - T_2) \qquad T_1 > T_2 \tag{1}$$

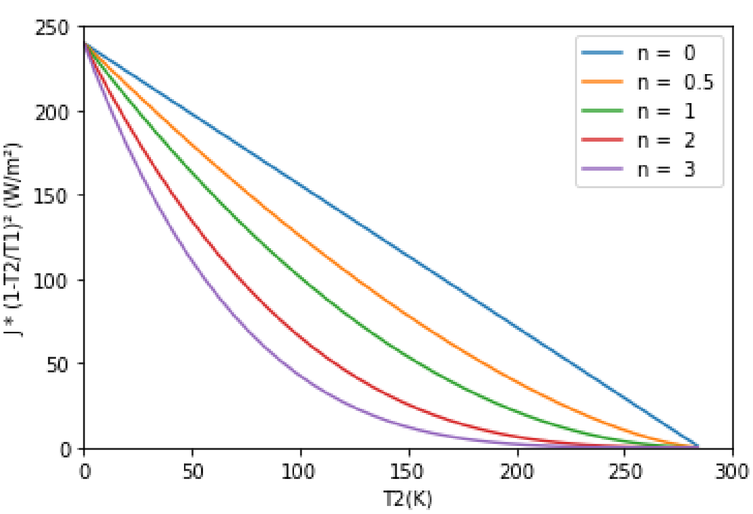
For generality let, $J/T_1 = k * (T_1 - T_2)^n$. Then, $(J/T_1) * (T_1 - T_2) = k * (T_1 - T_2)^{n+1}$ and

$$\ln(k) = \ln(J) - \ln(T_1) - n \ln(T_1 - T_2) \tag{2}$$

For linear dissipation, $n=1$. Upon differentiation with k and T_2 held constant,

$$0 = \frac{\delta J}{J} - \frac{\delta T_1}{T_1} - n \frac{\delta T_1}{T_1 - T_2} \quad ; \quad \left(\frac{\partial J}{\partial T_1}\right)_{T_2} = J * \left[\frac{1}{T_1} + \frac{n}{T_1 - T_2}\right] \tag{3}$$

This partial derivative is the system's T_1 thermal sensitivity. For all cases, if $T_2 = T_1$, $W=0$; if $T_2 = 0$, $W=J$. Exponential plots connecting these points for $J = 240$ *watts/m*^{2} and $T_1 = 285K$ show



T_1 sensitivities for $T_2 = 220K$ are:

n	0.0	0.5	1	2	3
$W/m^2/K$	0.84	2.69	4.53	8.23	11.92
T_1 Warming	4.40K	1.38K	0.82K	0.45K	0.31K

Earlier work, based on arbitrary profiles for radiative and convective fluxes between boundaries of varied temperature and flux, was summarized by the equation[1]

$$\left(\frac{\partial J_1}{\partial T_1}\right)_{T_2} = \gamma \frac{J}{T_1 - T_2} \quad ; \quad \gamma = 1.22 \pm 0.04 \tag{4}$$

Writing *Eq. 3* as

$$\left(\frac{\partial J}{\partial T_1}\right)_{T_2} = \frac{J}{T_1 - T_2} * \left[\frac{T_1 - T_2}{T_1} + n\right] \tag{5}$$

With our default values $T_1=285K$ and $T_2=220K$, the first bracketed term equals *0.2281,* suggesting that $n=1$ and linear dissipation are rather good approximations.

Many consider temperature a physical property. But it can not be reduced to a *m-l-t* function such as density. Its thermodynamic definition is as an integrating factor rendering $\delta S = \delta U/T$ an exact differential equation. Exact differential equations have a special property – solutions are path-independent. Physical states are time-reversible and theoretically past and future can be deduced from the present. Thermodynamic states can not. The necessary information has been irreversibly subsumed as entropy. At present, thermodynamics is restricted to path-independent modeling.

Equation 1 has an unusual property. Hypothetically, imagine a one-dimensional cell with boundary temperatures *100K* and *0K* and a flux of *100W.* We calculate a dissipation of *100W.* At some intermediate point, say *50K,* we visualize a junction of two tandem cells. For the first, we calculate *50W* dissipation. For the second we have *100W* flowing from *50K* to *0K* and 100W dissipation, for a combined total of *150W.* The resolution of this paradox is that only free energy can be dissipated. At the first boundary, all entering energy is considered free. At at the intermediate position only *50%* of the *100K* flux remains free and consequently only *50W* remains to be dissipated.

The table above also shows the warming required to increase flux $3.7\ W/m^2$. *IPCC* values are only consistent with $n=0$ models for which $T_1 - T_2$ is constant. If the functions plotted are to be symmetric about $T_2 = T_1$, odd integers for n are implied. It should be noted our derivation does not involve radiation or convection or any of the factors complicating *IPCC* models. It involves only three parameters which can be determined experimentally and does not require calculation as to what determines a value for J given T_1 and T_2. *Dissipation* is the antithesis of *Equilibrium*, the cornerstone of *RCE* models.

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1 https://pdquondam.net/Adiabatic_Lapse_Rate.pdf