

A Note on Convective Equilibrium

“It's not even wrong”

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I've often puzzled over the treatment of convection in climate modelings. The basic *Radiative Convective Equilibrium (RCE)* model supposes two parallel processes for transporting energy from the earth's surface. Should radiation be reduced by increased infra-red absorption, both radiation and convection might be expected to share in picking up the slack, their relative contributions a matter of some theoretical interest.

The *a priori* assumption underlying climate models is the *Adiabatic Lapse Rate (ALR)*, an equilibrium function wholly of a humidity-dependent heat capacity. Although rarely attributed, the hypothesis that this isentropic function might explain observed thermal gradients in the troposphere originated with none other than Lord Kelvin. At the January 1862 meeting of the Manchester Literary and Philosophical Society, a paper, ***On the Convective Equilibrium of Temperature in the Atmosphere, by Professor Wm. Thomson was read by Dr. Joule:***

“When all the parts of a fluid are freely interchanged and not subject to the influence of radiation and conduction, the temperature of the fluid is said by the Author to be in a state of convective equilibrium.”

In May 1866, J.C. Maxwell responds: *On the Dynamical Theory of Gases*, Philosophical Transactions of the Royal Society of London, Vol. 157, p. 86 (1867),

“The left-hand side of equation (147), as sent to the Royal Society, contained a term, the result of which was to indicate that a column of air, when itself, would assume a temperature varying with the height, and greater above than below. The mistake arose from an error in equation (143). Equation (147), as now corrected, shows that the flow of heat depends on the variation of temperature only, and not on the direction of the variation of pressure. A vertical column would therefore, when in thermal equilibrium, have the same temperature throughout.”

In October 1875, Ludwig Boltzmann writes: *Über das Wärmegleichgewicht von Gasen, auf welche äußere Kräfte wirken*, Sitzungsberichte der Mathematisch-Naturwissenschaftlichen, Vol. 72-II, p. 443 (1876),

“Aus dieser Formel folgt, daß trotz der Wirksamkeit der äußeren Kräfte für die Richtung der Geschwindigkeit irgend eines der Moleküle jede Richtung im Raume gleich wahrscheinlich ist, ferner dass in jedem Raumelemente des Gases die schwindigkeitsvertheilung des Gases genau ebenso beschaffen ist, wie in einem Gase von gleicher Temperatur, auf das keine Aussenkräfte wirken. Der Effect der äusseren Kräfte besteht blos darin, dass sich die Dichte im Gase von Stelle zu Stelle verändert und zwar in einer Weise, welche schon aus der Hydrostatik bekannt ist.”¹

1. Google's translation:

“From this formula, it follows that in spite of the effectiveness of the external forces for the direction of the velocity of any of the molecules, each direction in space is equally probable, furthermore that in each space element of the gas the velocity distribution of the gas is exactly the same as in a gas of same temperature, on which no external forces act. The effect of the external forces consists merely in the fact that the density in the gas changes from place to place in a manner which is already known from hydrostatics.”

Presuming Messrs. Maxwell and Boltzmann yet retain some credibility regarding matters thermodynamic, how do we then interpret non-isothermal profiles? Obviously, as non-equilibria. Lacking external support, such states will relax towards equilibrium and forestalling relaxation takes work, the steady-state dissipation of free energy, from external sources. Radiative fluxes dissipate through absorption, convective fluxes through viscosity.

RCE models offer detailed calculations for radiative flux but assume convection is implicitly incorporated with the *ALR*. Conventional justification is of a thermally-insulated parcel, *e.g.* a balloon, suspended in a fluid with an *ALR* profile. The parcel is supposed in a state of neutral buoyancy and its static energy independent of altitude. But, translation at finite rates remains limited by viscous resistance. Our parcel has no preferred direction of motion and convective flux therefore nil.

Convection cannot be described as a perturbation of an equilibrium state. Rather, the *ALR* is a 'tipping point' for formation of convection cells via an atmospheric Rayleigh–Bénard instability. Formally, one may suppose that convective flux is a function of a thermal gradient and an empirical coefficient decreasing with altitude. Similarly, radiative flux is a function of this same gradient, but a coefficient increasing with altitude. Both fluxes are dissipative and the thermal profile is that minimizing their joint dissipation (or deviations from a constant value in a one dimensional model).

The fatal flaw in current climate modelings is the presumption that convection fixes an equilibrium thermal gradient within the troposphere independent of greenhouse gases, whilst these gases shift tropospheric temperatures equally. For thermodynamic steady states, temperature and its gradient are functions of both radiative and convective energy fluxes! A basic model (linear coefficients), assuming earth-like boundary values for flux and temperature, yields a sensitivity of 4.51 W/K/m² with only the surface temperature perturbed and 1.56 W/K/m² with both boundary temperatures equally perturbed.

That thermodynamic steady states are minimum dissipation systems cannot be over-stressed. Their solutions minimize gradients and consequently surface temperatures. For every kilogram of rain that falls, 2260 kJ of energy has been transported by convection into higher altitudes. Hypothetically, one might well expect that increasing the altitude of cloud formation is but another mechanism facilitating energy transport across the troposphere.

“Science is a bit like the joke about the drunk who is looking under a lamppost for a key that he has lost on the other side of the street, because that's where the light (radiation) is. It has no other choice.” (Noam Chomsky)